

### **Introduction**

This brief technical report provides excerpts from Reference 1 and additional information on the stress and measurement of stress for AgilOptics' membrane mirrors. Section 2 defines the theory behind the membrane response to applied voltage given membrane tension, membrane to actuator pad separation, and actuator pad area. Section 3 describes two membrane tension measurement approaches and the results we obtained for our standard mirror fabrication techniques. Section 4 provides results from Zygo interferometer measurements of two Zernike aberrations verifying the agreement of our modeling results (using measured membrane stress) with actual mirror measurements.

### **Membrane Mirror Theory<sup>1</sup>**

Steady-state deflection of a membrane mirror is governed by the Poisson equation given by:<sup>2-4</sup>

$$\nabla^2 Z = -\left(\frac{F(r)}{T}\right) \quad (1)$$

where: Z is membrane deflection, T is membrane tension per unit length, and F is force on the membrane at a point r. For an electrostatic DM, the force of an actuator pad on the membrane is given by:<sup>4</sup>

$$F_k = \frac{\epsilon \text{Area}_k V_k^2}{2D_{sep}^2} \quad (2)$$

where:

$F_k$  = force on membrane due to actuator pad  $k$

$\epsilon$  = permittivity of free space

$\text{Area}_k$  = area of actuator pad  $k$

$V$  = voltage applied to pad  $k$

$D_{sep}$  = membrane to pad array separation

The analytical solution to equation (1) given in Reference 4 describes the steady-state membrane deflection for an actuator in the center of the membrane. We have generalized this solution to describe membrane deflection due to an actuator located at an arbitrary location within the membrane radius (R). This solution is given by:

$$Z(r, k) = \frac{F_k}{2\pi T} \left( \ln\left(\frac{R}{S}\right) + \frac{1}{2S^2} (S^2 - r^2) \right), \quad 0 < r < S$$

$$Z(r, k) = \frac{F_k}{2\pi T} \left( \ln\left(\frac{R}{S}\right) \frac{\ln\left(\frac{R_k}{r}\right)}{\ln\left(\frac{R_k}{S}\right)} \right), \quad S < r < R \quad (3)$$

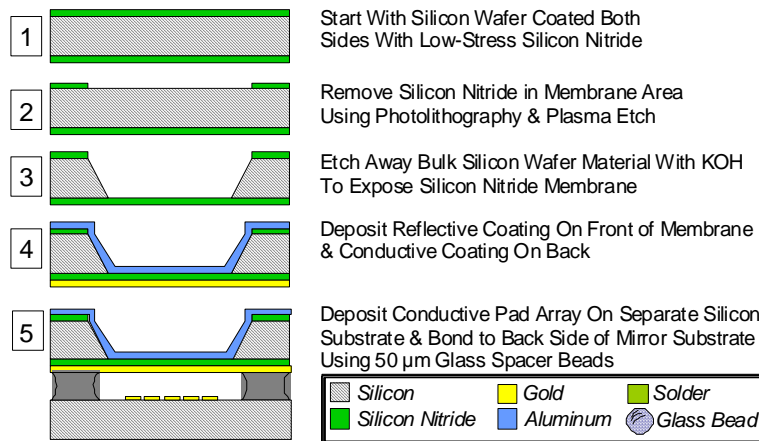
where:

$R_k$  = the distance of actuator  $k$  from membrane position  $r$

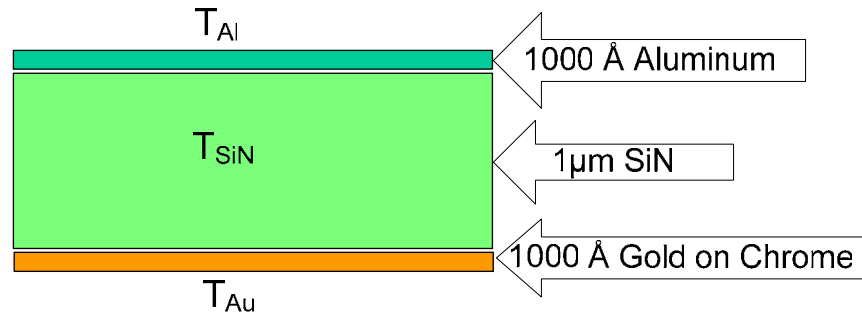
$S$  = actuator pad radius

### Membrane Tension Measurement

AgilOptics MEMS DMs are fabricated using bulk micro-machining techniques. The basic fabrication process is illustrated and described in Figure 1. Our standard mirror membranes generally have front-side and back-side metals deposited on the membrane as reflective and conductive surfaces, respectively. This is illustrated in Figure 2. The total membrane tension resulting from depositing these metals on the silicon-nitride membrane is the sum of the individual tension contributions from each layer. The mirror response to applied voltage depends on the total membrane tension as given by Equations 2 and 3. Thus, knowledge of the membrane tension is required to understand membrane mirror deflection.



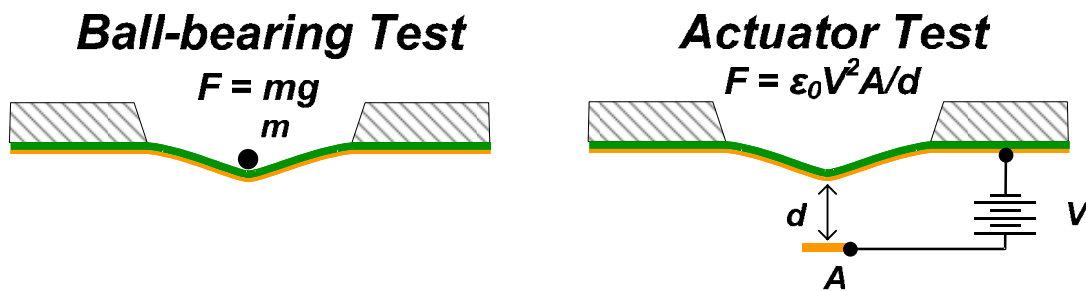
**Figure 1** Five-step bulk micro-machining process used by AgilOptics to fabricate membrane mirrors



$$T_{total} \stackrel{?}{=} T_{Al} + T_{SiN} + T_{Au}$$

**Figure 2** Metal coatings applied to silicon-nitride membrane contribute to total membrane tension

Membrane tension was measured using two different techniques (to provide increased confidence in the results) as depicted in Figure 3. In each case a force is applied to the membrane and the membrane deflection is measured using a Zygo interferometer. The first technique uses a ball bearing of known mass to apply a force resulting in a measurable membrane displacement. In the second technique, membrane deflection is measured for voltage applied to a single DM actuator. The force on the membrane is then calculated using known mirror parameters. With knowledge of membrane deflection and applied force, from each measurement, the membrane tension can be inferred. The calculated membrane stress based on these measurements is approximately 500 MPa. The silicon-nitride membrane material used in mirror fabrication has a measured stress of 50 to 100 Mpa. Thus, the major contributors to the total membrane tension, in this case, are the mirror coatings shown in Figure 2.

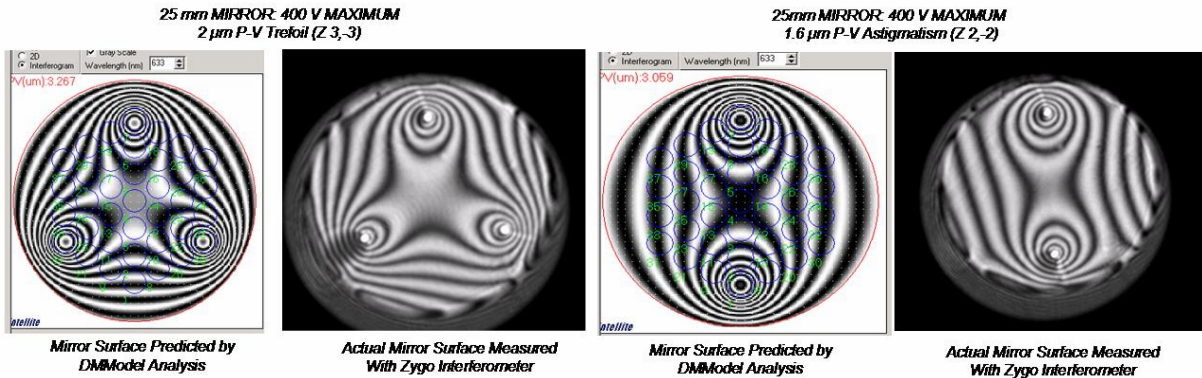


**Figure 3** Approaches used for membrane tension measurement

### ***Modeling Verification of Stress Measurements***

The membrane stress measurement can now be verified by using the measured value to model the complex mirror surface (according to the theory presented above) for given actuator voltages and comparing these results with mirror surface measurements for the same applied voltages. Figure 4 provides a comparison of mirror surfaces modeled using analytical influence functions (from AgilOptics' DMModel software) based on the membrane stress measurement (500 MPa) with Zygo interferometer measurements of the actual mirror surface. The mirror surfaces shown represent Zernike trefoil and astigmatism aberrations. Assuming the actuator forces add linearly, the modeled

surface is obtained by summing the influence functions from each actuator weighted by the individual actuator voltages. The modeled surfaces in Figure 4 show reasonably good agreement with the measurements, indicating that our modeled influence functions (based on the stress measurements) and the linear sum of influence functions provides a good approximation to the membrane surface deformation for these aberrations. Therefore, we expect that a stress of 500 MPa is accurate for our aluminum coated membrane mirrors.



**Figure 4** Modeled and measured mirror responses for Zernike Trefoil and Astigmatism aberrations

## Conclusions

The membrane stress of AgilOptics' aluminum coated deformable mirrors was measured using two different techniques to be approximately 500 MPa. Membrane mirror modeling using this stress value provides mirror surface deflections that show good agreement with mirror deflection measurements for the same applied DM actuator voltages. These measurement techniques and the resulting membrane stress measurements are accurate.

## References

1. K. Bush, D. German, B. Klemme, A. Marrs, M. Schoen, "Electrostatic Membrane Deformable Mirror Wavefront Control Systems: Design and Analysis," Proc. SPIE, Vol. 5553, (2004).
2. M. Yellin, "Using membrane mirrors in adaptive optics," Proc. SPIE, Vol. 75, (1976).
3. P. M. Morse, Vibration and Sound, McGraw-Hill, (1948).
4. R. P. Grosso and M. Yellin, "The membrane mirror as an adaptive optical element," J. Opt. Soc. A., Vol. 67, No. 3, March 1977.

## ***What Can Deformable Mirrors Do?***

- Correct Optical Aberrations
- Laser Beam Shaping
- Optical Image Enhancement

Deformable mirrors are revolutionizing laser and optical systems by replacing static components with dynamic optics. Deformable mirrors (DM) are adaptive optics with dynamic faces able to optimize or change the characteristics of reflected light for a specific application. With time-varying control, a DM can focus a beam at several different points at different times or it can replace a lens in an optical system. Deformable Mirrors can improve optical images in telescopes, cameras, and other imaging systems.

For further information and discussion about how deformable mirrors work and how they will solve your optical problems see the manuals for HVDD, Clarifi, and the application notes available on the Web.

[http://www.agiloptics.com/app\\_notes.htm](http://www.agiloptics.com/app_notes.htm)

<http://www.agiloptics.com/downloads.htm>